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Biased estimators in beta regression model in the presence of Multicollinearity

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Abstract

In regression modeling, the occurrence of a strong correlation among predictors has negative consequences for regression estimation. This problem can be solved using a variety of biased methods. From the generalized linear models, the beta regression model is a subset. When the response variable under examination is percentage, the beta regression model is a well-known model in research. Using various theories, a number of biased estimators for overcoming multicollinearity in beta regression models have been developed in the literature. There is a review of recent biased techniques for beta regression models. We can learn more about the performance of these biased estimators by comparing them.

Keywords: Multicollinearity; biased estimator; beta regression model; Monte Carlo simulation.

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المقدرون المتحيزون في نموذج الانحدار بيتا في ظل وجود علاقة خطية متعددة

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الملخص :

في نمذجة الانحدار، فإن حدوث ارتباط قوي بين المتنبئين له عواقب سلبية على تقدير الانحدار. يمكن حل هذه المشكلة باستخدام مجموعة متنوعة من الأساليب المتحيزة. من النماذج الخطية المعممة، يعتبر نموذج الانحدار بيتا مجموعة فرعية. عندما يكون متغير الاستجابة قيد الدراسة هو النسبة المئوية، فإن نموذج الانحدار بيتا هو نموذج معروف في مجال البحث. باستخدام نظريات مختلفة، تم تطوير عدد من المقدرين المتحيزين للتغلب على العلاقة الخطية المتعددة في نماذج الانحدار بيتا في الأدبيات. هناك مراجعة للتقنيات المتحيزة الحديثة لنماذج الانحدار بيتا. يمكننا معرفة المزيد عن أداء هؤلاء المقدرين المتحيزين من خلال مقارنتهم.

1. Introduction

In econometric modeling, the multicollinearity problem is a common problem. It shows that the explanatory variables have a strong relationship. In the case of severe multicollinearity, the covariance matrix of the ML estimator is well-known to be ill-conditioned. One of the negative consequences of this situation is that the regression estimates' variance becomes overstated. As a result, the coefficients' significance and magnitude are changed. Many traditional ways to solving this problem have been explored, such as deleting correlated variables, acquiring more data, or re-specifying the model.

The beta regression model is widely used in a variety of fields, including unemployment rates in certain countries, income distribution, the Gini index for each region, body fat percentage in medical disciplines, and graduation rates at key colleges. The beta regression model is a type of generalized linear model (GLM) that is used to determine the impact of specific explanatory variables on a non-normal response variable. The response component in beta regression, on the other hand, is limited to the range of zero to one, as in percentages, proportions, and fractions.

Shrinkage estimating approaches, such as ridge, Liu, and Liu-type estimations, have become a more widely accepted and successful methodology for solving the multicollinearity problem in a variety of regression models in recent years. The ridge estimator was proposed by Hoerl and Kennard (1970a, b). The ridge estimator works by applying a small definite amount (k) to the diagonal entries of the covariance matrix to boost conditioning, minimize MSE, and achieve consistent coefficients. For a review of ridge and Liu estimators in both linear and GLMs.

Ridge regression is a biased method that shrinks all regression coefficients toward zero to reduce the large variance [1]. This done by adding a positive amount to the diagonal of $\mathbf{X}^T\mathbf{X}$. As a result, the ridge estimator is biased but it guaranties a smaller mean squared error than the ML estimator.

In linear regression, the ridge estimator is defined as

$$\hat{\boldsymbol{\beta}}_{Ridge} = (\mathbf{X}^T\mathbf{X} + k\,\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y}, \tag{1}$$

where $\mathbf{y}$ is an $n \times 1$ vector of observations of the response variable, $\mathbf{X} = (\mathbf{x}_1, ..., \mathbf{x}_p)$ is an $n \times p$ known design matrix of explanatory variables, $\boldsymbol{\beta} = (\beta_1, ..., \beta_p)$ is a $p \times 1$ vector of unknown regression coefficients, $\mathbf{I}$ is the identity matrix with dimension $p \times p$, and $k \geq 0$ represents the ridge parameter (shrinkage parameter). The ridge parameter, k, controls the shrinkage of $\boldsymbol{\beta}$ toward zero. The OLS estimator can be considered as a special estimator from Eq. (1) with $k=0$. For larger value of k, the $\hat{\boldsymbol{\beta}}_{Ridge}$ estimator yields greater shrinkage approaching zero [2, 3].

2. Beta regression model

In beta regression model (BRM), the response variable, y, is assumed to follow beta distribution. The probability density function of beta distribution is given by

$$f\left(y;\theta_1,\theta_2\right)=\frac{\Gamma(\theta_2)}{\Gamma(\theta_1\theta_2)\Gamma((1-\theta_1)\theta_2)}\left(y\right)^{\theta_1\theta_2-1}\left(1-y\right)^{((1-\theta_1)\theta_2)-1},\quad 0<y<1, \qquad (2)$$

where $0<\theta_1<1$ and $\theta_2>0$. The mean and variance of Eq. (1) are given by, respectively, $E(y)=\theta_1$ and $V(y)=\theta_1(1-\theta_1)/(1+\theta_2)$ where θ_2 is a dispersion parameter. For a fixed value of θ_1, the $V(y)$ value decrease when the value of θ_2 increases.

Consider that we have a data set $\{(y_i,\mathbf{x}_i)\}_{i=1}^{n}$ where $y_i\in\mathbb{R}$ is a response variable belongs to Eq. (1), $\mathbf{x}_i=(x_{i1},x_{i2},...,x_{ip})\in\mathbb{R}^p$ is a $p\times1$ known explanatory variable vector, then in BRM, the mean is related to the explanatory variables as

$$g(\theta_{1i})=\mathbf{x}_i^T\boldsymbol{\beta}=\eta_i, \qquad (3)$$

where $\boldsymbol{\beta}=(\beta_0,\beta_1,...,\beta_p)$ is a $(p+1)\times1$ vector of unknown regression coefficients. Logit, probit, cloglog, and loglog are the used link functions of Eq. (2).

Ferrari and Cribari-Neto [4] extend the BRM to allow θ_2 to vary across observations. The BRM with varying dispersion (BRMVD) is defined as

$$g(\theta_{1i}) = \mathbf{x}_i^T \boldsymbol{\beta} = \eta_i \qquad (4)$$
$$h(\theta_{2i}) = \mathbf{s}_i^T \boldsymbol{\alpha} = \vartheta_i ,$$

where $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_k)$ is a $k \times 1$ vector of unknown regression coefficients and $\mathbf{s}_i = (s_{i1}, s_{i2}, ..., s_{ik}) \in \mathbb{R}^k$ is a $k \times 1$ known explanatory variable vector in addition to $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{ip})$ which are not exclusive, $p + k < n$.

The log-likelihood function of Eq. (3) is given by

$$\begin{aligned} \ell(\boldsymbol{\beta}, \boldsymbol{\alpha}) &= \sum_{i=1}^{n} \ell_i(\theta_{1i}, \theta_{2i}) \\ &= \ln \Gamma(\theta_{2i}) - \ln \Gamma((1-\theta_{1i})\theta_{2i}) + (\theta_{1i}\,\theta_{2i} - 1) \ln y_i \\ &\quad + \{((1-\theta_{1i})\theta_{2i}) - 1\} \ln\left(1 - y_i\right), \end{aligned} \qquad (5)$$

where $\theta_{1i} = g^{-1}(\eta_i)$ and $\theta_{2i} = h^{-1}(\vartheta_i)$. Differentiation of Eq. (4) with respect to the $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$, respectively, is defined as

$$\mathrm{U}_{\boldsymbol{\beta}}(\boldsymbol{\beta}, \boldsymbol{\alpha}) = \frac{\partial \ell(\boldsymbol{\beta}, \boldsymbol{\alpha})}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{n} \theta_{2i} (\tilde{y}_i - \tilde{\theta}_{1i}) \frac{\mathrm{d}\theta_{1i}}{\mathrm{d}\eta_i} \frac{\partial \eta_i}{\partial \beta_p}, \qquad (6)$$

$$\mathrm{U}_{\boldsymbol{\alpha}}(\boldsymbol{\beta}, \boldsymbol{\alpha}) = \frac{\partial \ell(\boldsymbol{\beta}, \boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}} = \sum_{i=1}^{n} \left\{ \begin{matrix} \theta_{1i} (\tilde{y}_i - \tilde{\theta}_{1i}) + \psi(\theta_{2i}) - \psi(1-\theta_{1i})\theta_{2i} \\ + \ln(1 - y_i) \end{matrix} \right\} \frac{\mathrm{d}\theta_{2i}}{\mathrm{d}\vartheta_i} \frac{\partial \vartheta_i}{\partial \alpha_k}, \qquad (7)$$

where $\tilde{y}_i = \ln(y_i / (1 - y_i))$, $\tilde{\theta}_{1i} = \psi(\theta_{1i}\theta_{2i}) - \psi((1-\theta_{1i})\theta_{2i})$, $\psi(.)$ represents the digamma function, $\mathrm{d}\theta_{1i} / \mathrm{d}\eta_i = 1/g'(\theta_{1i})$, and $\mathrm{d}\theta_{2i} / \mathrm{d}\vartheta_i = 1/h'(\theta_{2i})$. Then the maximum likelihood estimator of $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ are obtained from the solution of the nonlinear system $\mathrm{U}(\boldsymbol{\xi}) = 0$, where $\boldsymbol{\xi} = (\boldsymbol{\beta}^T, \boldsymbol{\alpha}^T)^T$ [5].

$$\hat{\boldsymbol{\beta}}_{MLE} = (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \hat{\mathbf{u}}, \qquad (8)$$

The ML estimator is asymptotically normally distributed with a covariance matrix that corresponds to the inverse of the Hessian matrix

$$\operatorname{cov}(\hat{\boldsymbol{\beta}}_{MLE}) = \left[-E\left(\frac{\partial^2 \ell(\boldsymbol{\beta})}{\partial \beta_i \, \partial \beta_k} \right) \right]^{-1} = \nu^{-1}(\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1}. \tag{9}$$

The mean squared error (MSE) of Eq. (7) can be obtained as

$$\begin{aligned} \operatorname{MSE}(\hat{\boldsymbol{\beta}}_{MLE}) &= E(\hat{\boldsymbol{\beta}}_{MLE} - \hat{\boldsymbol{\beta}})^T (\hat{\boldsymbol{\beta}}_{MLE} - \hat{\boldsymbol{\beta}}) \\ &= tr\, \nu^{-1}[(\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1}] \\ &= \nu^{-1} \sum_{j=1}^{p} \frac{1}{\lambda_j}, \end{aligned} \tag{10}$$

where λ_j is the eigenvalue of the $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ matrix. In the presence of multicollinearity, the matrix $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ becomes ill-conditioned leading to high variance and instability of the ML estimator of the beta regression parameters.

3. Ridge estimator

In the presence of multicollinearity, the matrix $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ becomes ill-conditioned leading to high variance and instability of the ML estimator of the BRM parameters. As a remedy, Månsson and Shukur [6] proposed the BR ridge estimator (BRR) as

$$\begin{aligned} \hat{\boldsymbol{\beta}}_{BRR} &= (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} \hat{\boldsymbol{\beta}}_{MLE} \\ &= (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \hat{\mathbf{v}}, \end{aligned} \tag{11}$$

where $k \geq 0$. The ML estimator can be considered as a special estimator from Eq. (10) with $k = 0$. Regardless of k value, the MSE of the $\hat{\boldsymbol{\beta}}_{BRR}$ is smaller than that of $\hat{\boldsymbol{\beta}}_{MLE}$ because the MSE of $\hat{\boldsymbol{\beta}}_{BRR}$ is equal to [7]

$$\text{MSE}(\hat{\boldsymbol{\beta}}_{BRR}) = \nu^{-1}\sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j + k)^2} + k^2\sum_{j=1}^{p}\frac{\alpha_j}{(\lambda_j + k)^2}, \tag{12}$$

where α_j is defined as the j^th^ element of $\gamma\hat{\boldsymbol{\beta}}_{MLE}$ and γ is the eigenvector of the $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}$ matrix. Comparing with the MSE of Eq. (11), $\text{MSE}(\hat{\boldsymbol{\beta}}_{BRR})$ is always small for $k > 0$.

4. Liu estimator

Another popular biased estimator which is known as Liu estimator has been adopted in Poisson regression model. The beta Liu estimator (BLE) is defined as

$$\hat{\boldsymbol{\beta}}_{BLE} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X} + \mathbf{I})^{-1}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X} + d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{MLE}, \tag{13}$$

where $0 < d < 1$. Regardless of d value, the MSE of the $\hat{\boldsymbol{\beta}}_{BLE}$ is smaller than that of $\hat{\boldsymbol{\beta}}_{MLE}$ because the MSE of $\hat{\boldsymbol{\beta}}_{BLE}$ is equal to [7]

$$\text{MSE}(\hat{\boldsymbol{\beta}}_{BLE}) = \nu^{-1}\sum_{j=1}^{p}\frac{(\lambda_j + d)^2}{\lambda_j(\lambda_j + 1)^2} + (d - 1)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j + 1)^2}. \tag{14}$$

5. Liu-type estimator

Alternative to Liu estimator, the Liu-type estimator was proposed by Liu [8] to overcome the problem of severe multicollinearity. The beta Liu-type estimator (BLT) is defined as

$$\hat{\boldsymbol{\beta}}_{BLT} = (\mathbf{X}^T \hat{\mathbf{W}}\mathbf{X} + k\ \mathbf{I})^{-1}(\mathbf{X}^T \hat{\mathbf{W}}\mathbf{X} - d\ \mathbf{I})\hat{\boldsymbol{\beta}}_{MLE}, \quad (15)$$

where $-\infty < d < \infty$ and $k \geq 0$. In Eq. (14), the parameter k can be used totally to control the conditioning of $\mathbf{X}^T \hat{\mathbf{W}}\mathbf{X} + k\ \mathbf{I}$. After the reduction of $\mathbf{X}^T \hat{\mathbf{W}}\mathbf{X} + k\ \mathbf{I}$ is reach a desirable level, then the expected bias that is generated can be corrected with the so-called bias correction parameter, d [9-13].

Liu [8] proved that, in terms of MSE, the Liu-type estimator has superior properties over ridge estimator. The MSE of $\hat{\boldsymbol{\beta}}_{BLT}$ is defined as

$$\text{MSE}(\hat{\boldsymbol{\beta}}_{BLT}) = \nu^{-1}\sum_{j=1}^{p}\frac{(\lambda_j - d)^2}{\lambda_j(\lambda_j + k)^2} + (d + k)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j + k)^2}. \quad (16)$$

6. Two-parameter estimator

Following Asar and Genç [14] and Huang and Yang [15] the two-parameter estimator in linear regression model is defined as:

$$\hat{\boldsymbol{\beta}}_{TPE} = (\mathbf{X}^T \mathbf{X} + k\ \mathbf{I})^{-1}(\mathbf{X}^T \mathbf{X} + k\ d\ \mathbf{I})\hat{\boldsymbol{\beta}}_{OLS}, \quad (17)$$

where $0 < d < 1$ and $k \geq 0$. For beta regression model, the two-parameter estimator (BTP) is defined as:

$$\hat{\boldsymbol{\beta}}_{BTP} = (\mathbf{X}^T \hat{\mathbf{W}}\mathbf{X} + k\ \mathbf{I})^{-1}(\mathbf{X}^T \hat{\mathbf{W}}\mathbf{X} + k\ d\ \mathbf{I})\hat{\boldsymbol{\beta}}_{BRM}. \quad (18)$$

It is obviously noted that the $\hat{\boldsymbol{\beta}}_{GTP}$ is a combination of two different estimators GRR and GLE. Furthermore, if $k = 1$, Eq. (18) will be the $\hat{\boldsymbol{\beta}}_{GLE}$ while if $k = 0$, Eq. (17) will be the $\hat{\boldsymbol{\beta}}_{GRM}$. Besides, when $d = 0$, then Eq. (17) will equal $\hat{\boldsymbol{\beta}}_{GRR}$.

In terms of MSE, the two-parameter estimator has superior properties over ML estimator. The MSE of $\hat{\boldsymbol{\beta}}_{BTP}$ is defined as

$$\text{MSE}(\hat{\boldsymbol{\beta}}_{BTP}) = v^{-1}\sum_{j=1}^{p+1}\left[\frac{(\lambda_j + kd)^2}{\lambda_j(\lambda_j + k)^2} + k^2(d-1)^2\frac{\alpha_j^2}{(\lambda_j + k)^2}\right]. \quad (19)$$

7. Monte Carlo Simulation Study

In this section, a Monte Carlo simulation experiment is used to examine the performance of our proposed estimator under different degrees of multicollinearity. The response variable of n observations from beta regression model is generated as $y_i \sim beta(\theta, v)$, where $v \in \{0.50, 1.5, 5\}$ and $\theta = \exp(\mathbf{x}_i^T\gamma)/(1+\exp(\mathbf{x}_i^T\gamma))$, $\gamma = (\gamma_1, ..., \gamma_p)$ with $\sum_{j=1}^{p}\gamma_j^2 = 1$ and $\gamma_1 = \gamma_2 = ... = \gamma_p$ [16]. The explanatory variables have been generated from the following: $x_{ij} = (1-\rho^2)^{1/2}z_{ij} + \rho z_{ip}$, $i = 1, 2, ..., n$, $j = 1, 2, ..., p$, where ρ represents the correlation between the explanatory variables and z_{ij}'s are independent standard normal pseudo-random numbers. Since, we are interested in the effect of multicollinearity, in which the degrees of correlation considered more important, then three values of the pairwise correlation are considered with $\rho = \{0.90, 0.95, 0.99\}$. In addition, an increase in the number of explanatory variables lead to an increase in MSE, then the number of the explanatory variables is considered as $p = 3$, $p = 7$, and $p = 15$. Further, three representative values of the sample size are considered: 50, 100, and 200 because the sample size has direct impact on the prediction accuracy.

For a combination of these different values of n, v, p, and ρ the generated data is repeated $R = 500$ times and the average MSE are determined.

The averaged MSE for all the combination of n, v, p, and ρ, are respectively summarized in Tables 1 to 3. The best value of the averaged MSE is highlighted in bold. According to the simulation results, we conclude the following:

1- Tables 1 to 3 show that two-parameter estimator, BTP, ranks first with respect to MSE. In the second rank, BLT estimator performs better than both MLE and BRR estimators. Additionally, MLE estimator has the worst performance among ridge, Liu, Liu-type, and two-parameter estimators which is significantly impacted by the multicollinearity.

2- Regarding the number of explanatory variables p, one can see that there is a negative impact on MSE, where there are increasing in MSE values when the p increasing from three variables to seven and fifteen variables.

3- However, in terms of precision parameter v, the MSE values are decreasing when v increasing. In addition, in terms of the sample size n, the MSE values decrease when n increases, regardless the value of ρ, v, and p.

Table 1: MSE values when $n = 50$

v	p	ρ	MLE	BRR	BLE	BLT	BTP
0.5	3	0.90	5.918	5.565	5.487	5.078	**5.0239**
		0.95	7.015	6.05	5.776	5.692	**5.6379**
		0.99	8.058	6.567	5.834	5.793	**5.7389**
	7	0.90	6.035	5.395	5.146	4.971	**4.9169**
		0.95	8.08	6.226	5.67	5.521	**5.4669**
		0.99	9.012	6.951	6.177	5.328	**5.2739**
	15	0.90	7.272	5.023	4.848	4.672	**4.6179**
		0.95	10.495	6.374	5.646	4.766	**4.7119**
		0.99	13.034	6.701	5.473	4.737	**4.6829**
1.5	3	0.90	5.61	5.257	5.179	4.77	**4.7159**
		0.95	6.707	5.742	5.468	5.384	**5.3299**
		0.99	7.75	6.26	5.526	5.485	**5.4309**

	7	0.90	5.727	5.087	4.838	4.663	**4.6089**
		0.95	7.771	5.918	5.362	5.213	**5.1589**
		0.99	8.704	6.643	5.869	5.02	**4.9659**
	15	0.90	6.964	4.715	4.54	4.364	**4.3099**
		0.95	10.187	6.066	5.338	4.458	**4.4039**
		0.99	12.726	6.393	5.165	4.43	**4.3759**
5	3	0.90	5.488	5.135	5.057	4.648	**4.5939**
		0.95	6.585	5.62	5.346	5.262	**5.2079**
		0.99	7.628	6.137	5.404	5.363	**5.3089**
	7	0.90	5.605	4.965	4.716	4.541	**4.4869**
		0.95	7.649	5.796	5.24	5.091	**5.0369**
		0.99	8.582	6.521	5.747	4.898	**4.8439**
	15	0.90	6.842	4.593	4.418	4.242	**4.1879**
		0.95	10.065	5.944	5.216	4.336	**4.2819**
		0.99	12.604	6.271	5.043	4.307	**4.2529**

Table 2: MSE values when $n = 100$

v	p	ρ	MLE	BRR	BLE	BLT	BTP
0.5	3	0.90	3.612	3.259	3.181	2.772	**2.7179**
		0.95	4.709	3.744	3.47	3.386	**3.3319**
		0.99	5.752	4.261	3.528	3.487	**3.4329**
	7	0.90	3.729	3.089	2.84	2.665	**2.6109**
		0.95	5.774	3.92	3.364	3.215	**3.1609**
		0.99	6.706	4.645	3.871	3.022	**2.9679**
	15	0.90	4.966	2.717	2.542	2.366	**2.3119**
		0.95	8.189	4.068	3.34	2.46	**2.4059**
		0.99	10.728	4.395	3.167	2.431	**2.3769**
1.5	3	0.90	3.304	2.951	2.873	2.464	**2.4099**
		0.95	4.401	3.436	3.162	3.078	**3.0239**
		0.99	5.444	3.954	3.22	3.179	**3.1249**
	7	0.90	3.421	2.781	2.532	2.357	**2.3029**
		0.95	5.465	3.612	3.056	2.907	**2.8529**
		0.99	6.398	4.337	3.563	2.714	**2.6599**
	15	0.90	4.658	2.409	2.234	2.058	**2.0039**
		0.95	7.881	3.76	3.032	2.152	**2.0979**
		0.99	10.42	4.087	2.859	2.124	**2.0699**
5	3	0.90	3.182	2.829	2.751	2.342	**2.2879**
		0.95	4.279	3.314	3.04	2.956	**2.9019**

	0.99	5.322	3.831	3.098	3.057	**3.0029**
7	0.90	3.299	2.659	2.41	2.235	**2.1809**
	0.95	5.343	3.49	2.934	2.785	**2.7309**
	0.99	6.276	4.215	3.441	2.592	**2.5379**
15	0.90	4.536	2.287	2.112	1.936	**1.8819**
	0.95	7.759	3.638	2.91	2.03	**1.9759**
	0.99	10.298	3.965	2.737	2.001	**1.9469**

Table 3: MSE values when $n = 200$

v	p	ρ	MLE	BRR	BLE	BLT	BTP
0.5	3	0.90	3.355	3.002	2.924	2.515	**2.4609**
		0.95	4.452	3.487	3.213	3.129	**3.0749**
		0.99	5.495	4.004	3.271	3.23	**3.1759**
	7	0.90	3.472	2.832	2.583	2.408	**2.3539**
		0.95	5.517	3.663	3.107	2.958	**2.9039**
		0.99	6.449	4.388	3.614	2.765	**2.7109**
	15	0.90	4.709	2.46	2.285	2.109	**2.0549**
		0.95	7.932	3.811	3.083	2.203	**2.1489**
		0.99	10.471	4.138	2.91	2.174	**2.1199**
1.5	3	0.90	3.047	2.694	2.616	2.207	**2.1529**
		0.95	4.144	3.179	2.905	2.821	**2.7669**
		0.99	5.187	3.697	2.963	2.922	**2.8679**
	7	0.90	3.164	2.524	2.275	2.1	**2.0459**
		0.95	5.208	3.355	2.799	2.65	**2.5959**
		0.99	6.141	4.08	3.306	2.457	**2.4029**
	15	0.90	4.401	2.152	1.977	1.801	**1.7469**
		0.95	7.624	3.503	2.775	1.895	**1.8409**
		0.99	10.163	3.83	2.602	1.867	**1.8129**
5	3	0.90	2.925	2.572	2.494	2.085	**2.0309**
		0.95	4.022	3.057	2.783	2.699	**2.6449**
		0.99	5.065	3.574	2.841	2.8	**2.7459**
	7	0.90	3.042	2.402	2.153	1.978	**1.9239**
		0.95	5.086	3.233	2.677	2.528	**2.4739**
		0.99	6.019	3.958	3.184	2.335	**2.2809**
	15	0.90	4.279	2.03	1.855	1.679	**1.6249**
		0.95	7.502	3.381	2.653	1.773	**1.7189**
		0.99	10.041	3.708	2.48	1.744	**1.6899**

8. Conclusions

In this study, we conducted a comprehensive assessment of the literature on biased estimators in beta regression models with multicollinearity. In terms of MSE, the two-parameter estimator performs better than MLE, BRR, BLT, and BLE in real-world applications. Finally, when there is multicollinearity in the beta regression model, the two-parameter estimator should be used.

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